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Nonlinear fuzzy modeling of a MCFC stack by an identification method

Fan Yang *, Xin-Jian Zhu, Guang-Yi Cao

Fuel Cell Research Institute, Shanghai Jiao Tong University, 1954 Huashan Road, Shanghai 200030, PR China

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Abstract

This paper reports a nonlinear fuzzy modeling study of a molten carbonate fuel cell (MCFC) stack by an identification method. MCFC is a complex nonlinear, multi-input and multi-output (MIMO) system that is hard to model by traditional methodologies. The Takagi–Sugeno (T–S) fuzzy model is suitable to model a large class of nonlinear MIMO system. In this paper, a MIMO T–S fuzzy model is used to represent MCFC. An identification method is used to determine both the nonlinear parameters of the antecedents and the linear parameters of the rules consequent in the T–S fuzzy model. The simulation tests reveal that obtained T–S fuzzy model using the identification method can efficiently approximate the static and dynamic behavior of a MCFC stack. Furthermore, based on this proposed T–S fuzzy model, valid control strategy studies such as predictive control, robust control can be developed.

Keywords: Modeling; Molten carbonate fuel cell (MCFC); Takagi–Sugeno (T–S) fuzzy model

1. Introduction

The molten carbonate fuel cell (MCFC) is an energy conversion device that produces electricity by the indirect combination of hydrogen and oxygen to water via an electron-carrying electrolyte. MCFC is one of the fuel cell technologies that have proven efficiency and environmental performance [1]. In the development of fuel cell technologies, the mathematical modeling is an important tool, which has the capability of predicting the fuel cell performance [2]. The results obtained from a reliable and effective model can be very useful to guide future research for fuel cell improvements and optimization. At the same time an effective model of the MCFC stack, is a prerequisite for control analysis and controller design.

As far as we known, the MCFC is a nonlinear multi-input and multi-output system. It is very difficult to model the MCFC system by the traditional methods. During the last several decades, various mathematical models have been established in the research on the internal mechanisms, ranging from a one-dimensional model to a three dimensional model [3–5]. These models are very useful for cell design and performance analysis. However, these models are too complicated to be used for control studies.

To meet the demands of developing valid control strategies, some researchers have attempted to establish novel fuel cell models by statistical data-driven approach. The artificial neural network (ANN) has been used to derive a solid oxide fuel cell (SOFC) model from the experimental data quickly [2]. Shen et al. [6] proposed a RBF neural networks identification technology to set up the nonlinear temperature model of MCFC stack. Jemei et al. [7] utilized ANN methodology for proton exchange membrane fuel cell (PEMFC) model. The AutoRegression with exogenous signal (ARX) identification algorithm was applied to computer linear SOFC system models [8]. The Hammerstein nonlinear system approach was used for identification of the SOFC model [9]. The support vector machine (SVM) was used to modeling study of the PEMFC and SOFC system [10,11]. However, most of modeling methods are in the point of steady-state models (*I–V* characteristics) or the single output, and these black-box models do not give much insight into the system being modeled.

Fuzzy models have proved to be useful in nonlinear dynamic system modeling. Several modeling methods based on fuzzy reasoning have been proposed in recent years, which can be roughly classified into the fuzzy relational model, the neural-network-based fuzzy model, the T–S fuzzy model and the

* Corresponding author. Tel.: +86 21 28164995; fax: +86 21 62932154.
E-mail address: yangfantracy@hotmail.com (F. Yang).

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Nomenclature

A_R	frequent factor of the resistance
A_{R_a}, A_D, A_E	coefficients of the cell inherent performance
$C_{a(c)}$	total concentration of anode (cathode)
C_p^s	heat capacity of the cell unit (J (kg K)$^{-1}$)
E	thermodynamic voltage (V)
E^0	reversible potential of the cell (V)
F	Faraday constant (96,485 C mol^{-1})
$F_a^{in}(F_c^{in})$	anode (cathode) total inlet mole flow (mol s^{-1})
$F_{a,i}^{in}(F_{c,i}^{in})$	molar flow rates of the ith reactant at the cell input of anode (cathode)
$F_{a,i}^{o}(F_{c,i}^{o})$	molar flow rates of the ith reactant at the cell output of anode (cathode)
$F_{a,i}^{r}(F_{c,i}^{r})$	reaction rate of the ith reactant of anode (cathode)
$h_{a(c)}$	channel height of anode (cathode) (m)
$\bar{h}_{a,i}^{in}(\bar{h}_{c,i}^{in})$	anode (cathode) inlet partial molar enthalpies (J mol^{-1})
$\bar{h}_i^o$	partial molar enthalpies at stack temperature (J mol^{-1})
ΔH_R, ΔH_a	activation energy of internal resistance, and ΔH_{O_2}, ΔH_{CO_2} anode and cathode gas
I	current (A)
J	current density (A m^{-2})
M^s	mass of the cell unit (kg)
N_{cell}	cell number
p	gas pressure
P_{dc}	stack dc power (W)
R	gas constant (8.3145 J (mol K^{-1}))
$R_{a,i}(R_{c,i})$	anode (cathode) total rate of production of species (mol s^{-1})
R_{ohmic}	ohmic resistance (Ω m^{-2})
s	cell active area (m^2)
T^o	stack solid average temperature (exit temperature) (K)
$T_a^{in}(T_c^{in})$	inlet temperature of anode (cathode) (K)
$u_f(u_{ox})$	fuel (oxidant) utilization
V_{dc}	cell voltage (V)
$x_{a,i}^{in}(x_{c,i}^{in})$	anode (cathode) inlet mole fractions
$x_{a,i}^{o}(x_{c,i}^{o})$	anode (cathode) outlet mole fractions
$Z_a(Z_c)$	impedance for electrode polarization of anode (cathode) (Ω m^{-2})

fuzzy basis function based model. Compared with other models, the T–S model needs less rules, each rule's consequence with linear function can describe the input–output mapping in a large range, and the fuzzy implication used in the model is also simple [12]. Takagi–Sugeno (T–S) fuzzy model has the ability to approximate a large class of static and dynamical MIMO nonlinear systems. There exist a large number of research papers on the application of model identification successfully and control strategies for T–S fuzzy model [13–15]. However, the concrete study of modeling multivariate MCFC using T–S fuzzy model cannot be found in prior papers.

In this paper, a nonlinear MIMO T–S fuzzy modeling of a molten carbonate fuel cell (MCFC) stack is built with an identification method. T–S fuzzy model consists of *if–then* rules with fuzzy antecedents and mathematical functions in the consequent part. Antecedent identification by which the MCFC inputs–outputs variable space is divided into many subspaces (rules) is implemented based on the principle of Fuzzy C-Means clustering method. In every rule, the consequent part parameters that make the nonlinear MCFC characteristic to be fitted by a linear model are identified by using the Kalman filtering algorithm. In the process of fuzzy modeling, a novel way is introduced to determine satisfactory number of rules. In our study, the proposed MIMO T–S fuzzy model of MCFC is built with data obtained from a physical model of 10 kW MCFC. The encouraging results from this study demonstrate that there is a potential to introduce the nonlinear fuzzy modeling to development of fuel cells by the data obtained from various fuel cell experiments in the future application.

2. MCFC dynamic physical model

The proposed model is based on the following assumptions:

(1) Stream mixture thermodynamic properties follow the ideal gas mixture law.
(2) A uniform gas distribution among cells is supposed.
(3) All cells have the same temperature and current density.
(4) Fuel processor dynamics are not considered.

2.1. Electrochemical model

The basic structure of a single cell is shown in Fig. 1. There are two main sets of chemical kinetics. This chemical kinetics are those associated with anode reactions, cathode reactions. In the following equations, R vector is used to represent rate (respectively) of five individual species, which are hydrogen, carbon dioxide, water, oxygen and nitrogen.

- anode: $H_2 + CO_3^{2-} \rightarrow CO_2 + H_2O + 2e^-$

$$R_a = \left[-\frac{N_{cell} I}{2F} \quad \frac{N_{cell} I}{2F} \quad \frac{N_{cell} I}{2F} \quad 0 \quad 0 \right]$$

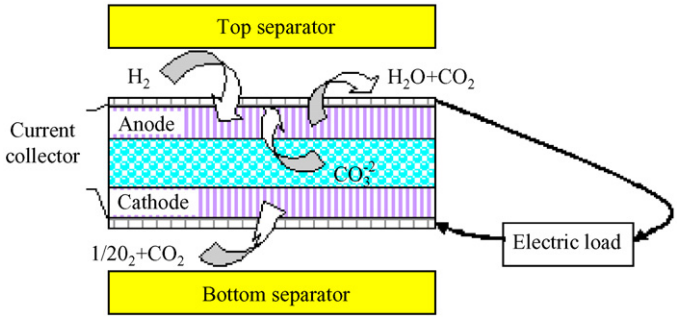

Fig. 1. Principle of electricity generating in a MCFC single cell.

$$I = Js \tag{1}$$

$$N_{\text{cell}} V_{\text{a}} C_{\text{a}} \frac{\mathrm{d}x^{\text{o}}_{\text{a},i}}{\mathrm{d}t} = F^{\text{in}}_{\text{a},i} - F^{\text{o}}_{\text{a},i} + F^{\text{r}}_{\text{a},i} = F^{\text{in}}_{\text{a}} x^{\text{in}}_{\text{a},i} - \left(F^{\text{in}}_{\text{a}} + \sum_{i=1}^{5} R_{\text{a},i} \right) x^{\text{o}}_{\text{a},i} + R_{\text{a},i} \tag{2}$$

- cathode: $2\text{e}^- + \text{CO}_2 + \frac{1}{2}\text{O}_2 \rightarrow \text{CO}_3{}^{2-}$

$$R_{\text{c}} = \begin{bmatrix} 0 & -\dfrac{N_{\text{cell}} I}{2F} & 0 & -\dfrac{N_{\text{cell}} I}{4F} & 0 \end{bmatrix}$$

$$N_{\text{cell}} V_{\text{c}} C_{\text{c}} \frac{\mathrm{d}x^{\text{o}}_{\text{c},i}}{\mathrm{d}t} = F^{\text{in}}_{\text{c},i} - F^{\text{o}}_{\text{c},i} + F^{\text{r}}_{\text{c},i} = F^{\text{in}}_{\text{c}} x^{\text{in}}_{\text{c},i} - \left(F^{\text{in}}_{\text{c}} + \sum_{i=1}^{5} R_{\text{c},i} \right) x^{\text{o}}_{\text{c},i} + R_{\text{c},i} \tag{3}$$

where using the ideal gas law, the exit stream total concentration becomes $C_{\text{a(c)}} = p_{\text{a(c)}}/RT^{\text{o}}$. The compartment volume $V_{\text{a(c)}}$ equals to $sh_{\text{a(c)}}$. The fuel utilization $u_{\text{f}} = N_{\text{cell}} Js/2FF^{\text{in}}_{\text{a}} x^{\text{in}}_{\text{a,H}_2}$.

2.2. Thermal model

The fuel cell power output is closely related to the temperature of the cell unit. Assuming that energy accumulates only in the stack solid mass, gas mixtures are ideal, and exit stream temperatures are equal to the solid stack temperature, the energy conservation equation is [16]:

$$M^{\text{s}} C^{\text{s}}_{p} \frac{\mathrm{d}T^{\text{o}}}{\mathrm{d}t} = F^{\text{in}}_{\text{a}} \left[\sum_{i=1}^{5} x^{\text{in}}_{\text{a},i} (\bar{h}^{\text{in}}_{\text{a},i} - \bar{h}^{\text{o}}_{i}) \right] - \sum_{i=1}^{5} \bar{h}^{\text{o}}_{i} R_{\text{a},i} + F^{\text{in}}_{\text{c}} \left[\sum_{i=1}^{5} x^{\text{in}}_{\text{c},i} (\bar{h}^{\text{in}}_{\text{c},i} - \bar{h}^{\text{o}}_{i}) \right] - \sum_{i=1}^{5} \bar{h}^{\text{o}}_{i} R_{\text{c},i} - Q_{\text{loss}} - P_{\text{dc}} \tag{4}$$

Under the ideal gas supposition, the partial molar enthalpies are calculated by the formulation:

$$\bar{h}_i = \bar{h}^{\text{ref}}_{i} + \int_{T_{\text{ref}}}^{T} c_{p,i}(u)\,\mathrm{d}u \tag{5}$$

And the coefficients of the specific heats $c_{p,i}$ are encountered in standard reference tables:

$$c_{p,i} = a_i + b_i T + c_i T^2 + d_i T^3 + e_i T^4 \tag{6}$$

2.3. Operating cell voltage

The empirical relationships developed in previously reported work [17] are employed to estimate a voltage drop in a fuel cell.

The cell-unit performance of the electric power generation is expressed:

$$V_{\text{dc}} = E - (R_{\text{ohmic}} - Z_{\text{a}} - Z_{\text{c}})J \tag{7}$$

where E is given by the Nernst potential:

$$E = E^0 + \frac{RT}{2F} \ln \frac{p_{\text{H}_2,\text{a}} p^{1/2}_{\text{O}_2,\text{c}} p_{\text{CO}_2,\text{c}}}{p_{\text{H}_2\text{O},\text{a}} p_{\text{CO}_2,\text{a}}} \tag{8}$$

and

$$E^0 = \frac{4184 \times [58.3 - (0.0113 + 9.6 \times 10^{-7} T)T]}{2F} \tag{9}$$

$$R_{\text{ohmic}} = A_R \exp\left(-\frac{\Delta H_R}{RT} \right) \tag{10}$$

$$Z_{\text{a}} = A_{R_{\text{a}}} T \exp\left(-\frac{\Delta H_{\text{a}}}{RT} \right) p^{-0.5}_{\text{H}_{2,\text{a}}} \tag{11}$$

$$Z_{\text{c}} = A_{\text{D}} T \exp\left(-\frac{\Delta H_{\text{O}_2}}{RT} \right) p^{-0.75}_{\text{O}_2} p^{0.5}_{\text{CO}_2} + A_{\text{E}} T \exp\left(-\frac{\Delta H_{\text{CO}_2}}{RT} \right) p^{-1.0}_{\text{CO}_2} \tag{12}$$

In Eqs. (8)–(12), T is the arithmetic average of cathode inlet and exit temperature. Anode and cathode partial pressures are arithmetic averages of inlet and exit gas partial pressures:

$$T = \frac{T^{\text{in}}_{\text{c}} + T^{\text{o}}}{2} \tag{13}$$

$$p_{\text{a(c)},i} = \frac{x^{\text{in}}_{\text{a(c)},i} + x^{\text{o}}_{\text{a(c)},i}}{2p_{\text{a(c)}}/p_{\text{atm}}} \tag{14}$$

3. T–S fuzzy modeling

3.1. Problem formulation

We consider a MCFC system $G(U, Y)$ as a MIMO system, let $u(u \in U \subseteq \mathbb{R}^r)$ be input variable and $y(y \in Y \subseteq \mathbb{R}^q)$ output variable. For the MIMO system, it can be divided into q multi-input and single output (MISO) system, and each of MISO can be fit by a fuzzy T–S model independently [14]. We define:

$$\{u_j(k)\}_0^{n_{u_j}} = [u_j(k), \ldots, u_j(k - n_{u_j} + 1)] \quad (j = 1, \ldots, r),$$

$$\{y_l(k)\}_0^{n_{y_l}} = [y_l(k), \ldots, y_l(k - n_{u_l} + 1)] \quad (l = 1, \ldots, q)$$

with n_{u_j} and n_{y_l} are the order of u_j and y_l, respectively, then each MISO subsystem can be denoted as follows:

$$y_l(k+1) = f(x(k)) \quad (l = 1, \ldots, q) \tag{15}$$

with $x(k) = [\{u_1(k)\}_0^{n_{u_1}}, \ldots, \{u_r(k)\}_0^{n_{u_r}}, \{y_1(k)\}_0^{n_{y_1}}, \ldots, \{y_q(k)\}_0^{n_{y_q}}] = [x_{k_1}, \ldots, x_{k_n}]$ is the regression data vector consisting of input–output data at the kth instant and before.

The T–S fuzzy model with linear consequents employed to fit the MISO subsystem in this paper is a collection of fuzzy rules, which is in the form of "*If . . . then . . .*" The *i*th rule of the *l*th output $\hat{y}_{l,i}(k+1)$ is given by:

$R_{l,i}$: *If* $x(k)$ is A_i^l, *then*

$$\hat{y}_{l,i}(k+1) = p_{i,0}^l + p_{i,1}^l x_{k_1} + \cdots + p_{i,n}^l x_{k_n} \quad (i = 1, \ldots, c) \tag{16}$$

where c is the number of rules, and $A_i^l = \{A_{i,1}^l, \ldots, A_{i,n}^l\}$ is the set of membership functions associated to the *i*th rule. $p_i^l = [p_{i,0}^l, p_{i,1}^l, \ldots, p_{i,n}^l]$ is the parameter vector of the *i*th submodel (rule).

If the method of product fuzzy inference and weighted mean is employed, the output of the T–S fuzzy model is inferred as follows:

$$\hat{y}_l(k+1) = \frac{\sum_{i=1}^c \mu_i(k)\hat{y}_{l,i}(k+1)}{\sum_{i=1}^c \mu_i(k)} \tag{17}$$

$$\mu_i(k) = \prod_{j=1}^n \mu_{i,j}(k) \tag{18}$$

where $\mu_{i,j}(k)$ is the match degree of the component x_{kj} with respect to $A_{i,k}^l$. $\mu_i(k)$ is the match degree of $x(k)$ with respect to A_i^l, namely the membership degree of $x(k)$ in the *i*th rule.

The antecedent part of "*If . . .*" in the form of fuzzy set, is equal to fuzzy partition of data space of the MCFC plant, and the consequent part of "*then . . .*" represented by a linear functional relation, is a linear composition of input–output variables. The identification problem that we want to solve is to find the fuzzy MCFC model of reasonable complexity that minimizes the error variance between the fuzzy MCFC model output and the real MCFC stack output. In this model, what is to be partitioned is the whole space spanned by regression data vectors. We separate the identification of the fuzzy model into two parts—antecedent and consequent identification. All the subsystems ($y_l(k+1) = f(x(k))$, $l = 1, \ldots, q$) can be identified independently in the same way, so the subscription l is omitted in the following description.

3.2. Antecedent if-part identification

Antecedent identification is implemented by fuzzy clustering based on the principle of Fuzzy C-Means (FCM) algorithm. Let $\{x_k: k = 1, \ldots, N\}$ be a set of N sample data from the real plant. Each sample $x_k = [x_{k_1}, x_{k_2}, \ldots, x_{k_n}]$ has n components.

The FCM algorithm was introduced by Bezdek [18]. In this paper, FCM algorithm is used to partition the collection of N data from real MCFC stack into c fuzzy clusters. In the fuzzy modeling, the c means the number of fuzzy rules. The idea of FCM is using the weights that minimize the total weighted mean-square error:

$$J(\mu, V) = \sum_{k=1}^N \sum_{i=1}^c (\mu_{i,k})^m d_{ik}^2 = \sum_{k=1}^N \sum_{i=1}^c (\mu_{i,k})^m ||x_k - v_i||^2 \tag{19}$$

where $\mu_{i,k}$ is the membership degree of the *k*th sample in the *i*th cluster. μ is the matrix $\mu = \{\mu_{i,k}\}$ which satisfies: $\mu_{i,k} \in [0, 1]$, $\sum_{i=1}^c \mu_{i,k} = 1$. The exponent m is a real number greater than unity. $V = [v_1, v_2, \ldots, v_c]$ represents the center of each cluster. The d_{ik} is the distance from x_k to v_i. Local minimization of the function J is accomplished by repeatedly adjusting the values of $\mu_{i,k}$ and v_i according to the following relation:

$$\mu_{i,k} = \frac{1}{\sum_{j=1}^c (d_{ik}/d_{jk})^{2/m-1}} \tag{20}$$

$$v_i = \frac{\sum_{k=1}^N \mu_{i,k}^m x_k}{\sum_{k=1}^N \mu_{i,k}^m} \tag{21}$$

3.3. Consequent then-part identification

The consequent part of the fuzzy rule is identified by using the Kalman filtering algorithm.

Given the data x_k where k represents the *k*th sampling, the $(k+1)$th predictive output $\hat{y}(k+1)$ of fuzzy model, can be obtained by formulation (17). Let $\beta_{ki} = \mu_i(k)/\sum_{i=1}^c \mu_i(k)$, then $\hat{y}(k+1)$ can be written as:

$$\hat{y}(k+1) = \sum_i^c \beta_{ki} \, (p_{i_0} + p_{i_1} x_{k_1} + \cdots + p_{i_n} x_{k_n}) \tag{22}$$

While $x_k = [x_{k_1}, x_{k_2}, \ldots, x_{k_n}]$ $(k = 1, 2, \ldots, N)$ represent the data of n dimension input variables of MCFC, and $Y = [y_1, y_2, \ldots, y_N]'$ are the outputs, and the $\beta_k = (\beta_{k_1}, \beta_{k_2}, \ldots, \beta_{k_c})$ can be calculated. The outputs can be expressed from (22) using the matrix form as follows:

$$Y = hP \tag{23}$$

$$h = \begin{bmatrix} \beta_{11} \ldots & \beta_{1c} \ldots & x_{11}\beta_{11} \ldots & x_{11}\beta_{1c} \ldots & x_{1n}\beta_{11} \ldots & x_{1n}\beta_{1c} \\ \beta_{21} \ldots & \beta_{2c} \ldots & x_{21}\beta_{21} \ldots & x_{21}\beta_{2c} \ldots & x_{2n}\beta_{21} \ldots & x_{2n}\beta_{2c} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \beta_{N1} \ldots & \beta_{Nc} \ldots & x_{N1}\beta_{N1} \ldots & x_{N1}\beta_{Nc} \ldots & x_{Nn}\beta_{N1} \ldots & x_{Nn}\beta_{Nc} \end{bmatrix}_{N \times c(n+1)} \tag{24}$$

$$P = [p_{10}, p_{20}, \ldots, p_{c0}, p_{11}, p_{21}, \ldots, p_{c1}, \ldots, p_{1n}, p_{2n}, \ldots, p_{cn}]'_{c(n+1)\times 1} \tag{25}$$

The consequent parameter vector P can be estimate by Eq. (23). To void the computationally expensive matrix inversion

operation, the Kalman filtering algorithm is applied in this paper. The Kalman filtering algorithm enables calculation of the new, adapted values of parameters of the antecedent part, on the basis of the new sample and the known parameter values [19]. Here, we apply it to calculate the parameter vector P as follows:

$$\hat{P}_{k+1} = \hat{P}_k + \frac{S_{k+1}h'_{k+1}(y_{k+1} - h_{k+1}\hat{P}_k)}{Q + h_{k+1}S_k h'_{k+1}} \tag{26}$$

$$S_{k+1} = S_k - \frac{S_k h'_{k+1} h_{k+1} S_k}{Q + h_{k+1}S_k h'_{k+1}} \tag{27}$$

$$k = 0, 1, 2, \ldots, N-1 \tag{28}$$

where $\hat{P}_k$ is the estimated parameter vector. $\hat{P}_0$ is zero vector. The h_{k+1} is the $(k+1)$th row of matrix h, and y_{k+1} is the real output of $(k+1)$th sample point. $S_0 = \alpha I$ (I is an identity matrix and α is a large positive number), and $Q = \exp(-N/z)$ (N represents the iteration counter and z is a constant).

3.4. Fuzzy modeling procedure

The number of fuzzy rules is an important factor that affects the performance of a fuzzy model. The choice of an appropriate number of rules is crucial to the design of fuzzy systems because practical considerations usually need to keep the number of rules as low as possible to reduce complexity. While too many redundant rules result in a complex fuzzy model and increase difficulties of implement, too few rules produce the fuzzy model cannot approximate a real plant. In the paper, the cluster number c in FCM algorithm corresponds to the number of "*If*... *then*" rules. However, for many kinds of clustering methods, including FCM algorithm, clustering number c is always needed in advance. Here, a novel way [15] to determine the cluster number (namely the number of fuzzy rules) is introduced to the fuzzy modeling. Let the clustering method start with $c = 2$. Then determine whether a new cluster center should be increased or not by evaluating modeling results. Root mean squared error (RMSE) is employed here to evaluate modeling results:

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(\hat{y}_k - y_k)^2} \tag{29}$$

where N is the number of sample data from the real MCFC stack, $\hat{y}_k$ is the predictive output of fuzzy model and y_k is the output of the real MCFC stack. If the modeling result is not satisfied, from the given sample data, a sample x_k that is most different from the existing cluster centers $V = [v_1, v_2, \ldots, v_c]$ can be fined as a new center v_{c+1}. Where the following definition for v_{c+1}:

$$v_{c+1} = x_k \left(k = \arg_k \min \sum_{\substack{1 \le i,\, j \le c \\ i \ne j}} (\mu_{ik} - \mu_{jk}) \right) \tag{30}$$

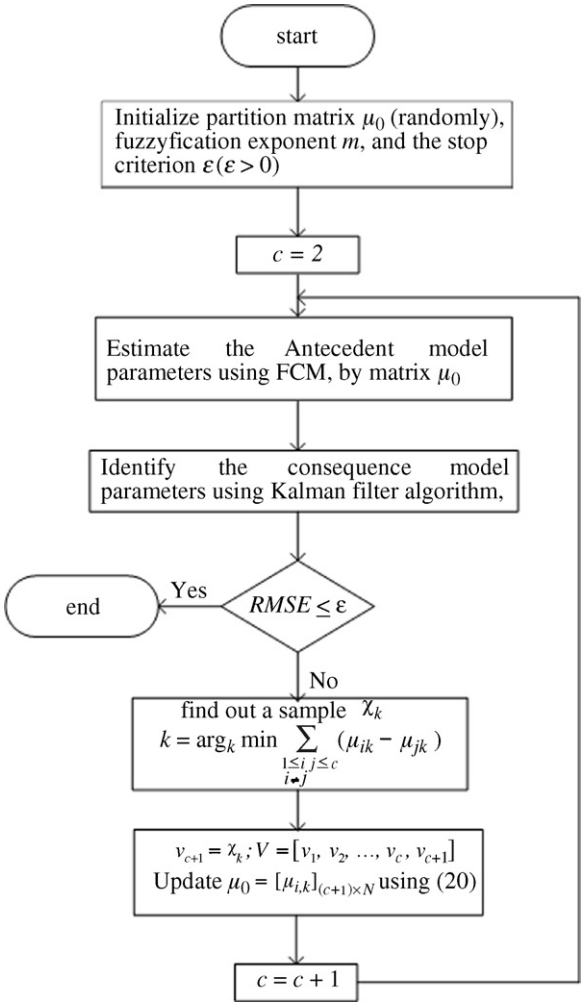

Fig. 2. Flowchart of the fuzzy modeling method.

Start with $V = [v_1, v_2, \ldots, v_{c+1}]$ as initial cluster centers, and compute the new not-random partition matrix μ_0. Then FCM algorithm is applied to divide the sample data into $c+1$ parts again. Do the above steps again until the result is satisfactory. Then the T–S fuzzy model can be obtained by the antecedent and consequent identification methods. The basic design steps of the modeling method are depicted in Fig. 2.

4. Results

As we know, output voltage and the temperature are the core of any fuel cell modeling. For a given MIMO MCFC stack, the output voltage and the temperature are influenced by many operating parameters such as current density, temperature, pressure, fuel utilization (u_f), oxidant utilization (u_{ox}), and gas flows, etc. However, in practice, the gas flows must vary within the allowable gas utilization range considering the safe operating areas of a plant. Up to now, no model has ever been able to accommodate all these operating parameters. Our T–S model is no exception. In our experiment, the MCFC can be regarded as a system with two-input (u_f, u_{ox}) and two-output (V_{dc}, T^o), and current density J is considered to be a disturbance and other

operating parameters are held constant. Define the following input–output relations:

$$\hat{y}_{V_{dc}}(k+1) = f(u_f(k), u_{ox}(k), J(k), y_{V_{dc}}(k)) \quad (31)$$

$$\hat{y}_{T^o}(k+1) = f(u_f(k), u_{ox}(k), J(k), y_{T^o}(k)) \quad (32)$$

where $\hat{y}_{V_{dc}}$, $\hat{y}_{T^o}$ are the predictive outputs of fuzzy model and $y_{V_{dc}}$, y_{T^o} are the outputs from the real MCFC stack. The order of inputs and outputs can be increased according to a concrete system. This above simplification does not impair the validity of our study, which is aimed at modeling MCFC by the proposed method in this paper. The proposed fuzzy modeling method is used to identify a T–S fuzzy model of MCFC based on simulation data.

4.1. Preparing simulation data

The 10 kW MCFC stack is used in the simulation. The stack consists of 25 cells with the anode and cathode in co-flow. The effective electrolyte area of the cell is 0.4 m^2, and the electrolyte tile between anode and cathode consists of mixed carbonate of Li_2CO_3 and K_2CO_3. The compositions of fuel and oxidant are set at constant values. The dynamic physical model as shown in Section 2 replaces the real MCFC stack to generate the simulation data required for the identification of the T–S fuzzy model. The parameters of this fuel cell are given in Table 1. The data sources blocks developed in MATLAB based on Section 2 is shown in Fig. 3. For the purpose of identification, the dynamic physical model is excited with uniformly random input signals included the fuel utilization (40–90%), the oxidant utilization (10–80%), and the current density (100–1800 A m^{-2}). To obtain values at integer time points, the fourth-order Runge–Kutta method was used to find the numerical solution to the dynamic physical model in the simulation. A set of 10,000 data was collected from the simulation. The first 6000 data were used for the identification of a T–S fuzzy model of the fuel cell, while the remaining 4000 data were used for validation purposes.

Table 1
Parameters of the MCFC stack used in the fuzzy modeling

Parameter	Unit	Value
N_{cell}		25
P_{dc}	kW	10
J	A m^{-2}	100–1800
V_{dc}	V	Variable
u_f		0.4–0.9
u_{ox}		0.1–0.8
s	m^2	0.4
h_a	m	1.2×10^{-3}
h_c	m	2×10^{-3}
C_p^s	kg m^{-3}	7900
A_R		6.43×10^{-2}
ΔH_R	kJ mol^{-1}	−25.5
A_{R_a}		8.11×10^{-9}
ΔH_a	kJ mol^{-1}	−74.4
A_D		2.10×10^{-10}
ΔH_{O_2}	kJ mol^{-1}	−83.4
A_E		1.58×10^{-5}
ΔH_{CO_2}	kJ mol^{-1}	−7.12
T_a^{in}	K	873
T_c^{in}	K	823
x_{a,H_2}^{in}	Mole fraction	0.64
x_{a,CO_2}^{in}	Mole fraction	0.2
x_{a,H_2O}^{in}	Mole fraction	0.16
x_{c,CO_2}^{in}	Mole fraction	0.3
x_{c,N_2}^{in}	Mole fraction	0.553
x_{c,O_2}^{in}	Mole fraction	0.147

4.2. Predicting with the MIMO T–S fuzzy model

The parameters pre-specified in the modeling method as shown in Section 3.4 are the exponent $m = 2$, stop criterion

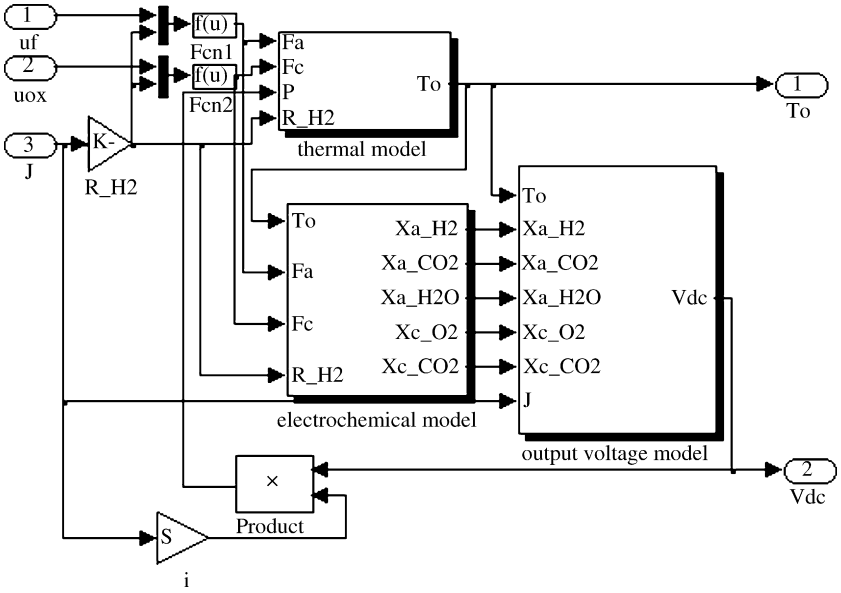

Fig. 3. Data sources of T–S fuzzy modeling.

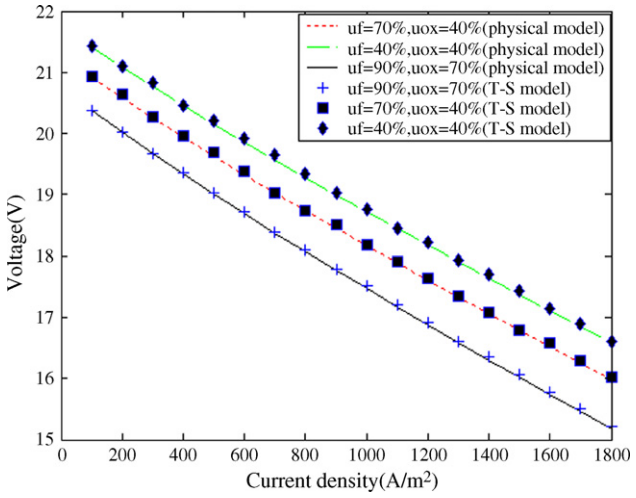

Fig. 4. J–V_{dc} chart.

$\varepsilon = 0.15$, and $\alpha = 10^9$. After fuzzy modeling of Section 3.4, a T–S fuzzy model is obtained, which can be used to predict new input data. The estimated optimal number of fuzzy rules is equal to 12. The RMSE of output voltage obtained in training and test process is 0.1028 and 0.1172 respectively. The RMSE of temperature obtained in training and test process is 0.1249 and 0.1283 respectively. The static J–V_{dc} characteristics generated by the T–S fuzzy model showed good consistency with the physical model under various utilization, as can be seen in Fig. 4. Now the T–S fuzzy model is used to predict the dynamic characteristics of the physical model. The step changes in the stack current density (from 1000 A m^{-2} to 1500 A m^{-2}) and fuel utilization (from 80% to 85%) are applied with 50% oxidant utilization. The comparison between predicted (T–S model) and experimental (physical model) output voltage dynamic curves is represented in Fig. 5. At the same time, the predicted and experimental temperatures are shown in Fig. 6. From Figs. 4–6, we can see the obtained MIMO T–S fuzzy model can approximate the static and dynamic behavior of the physical MCFC model with good accuracy.

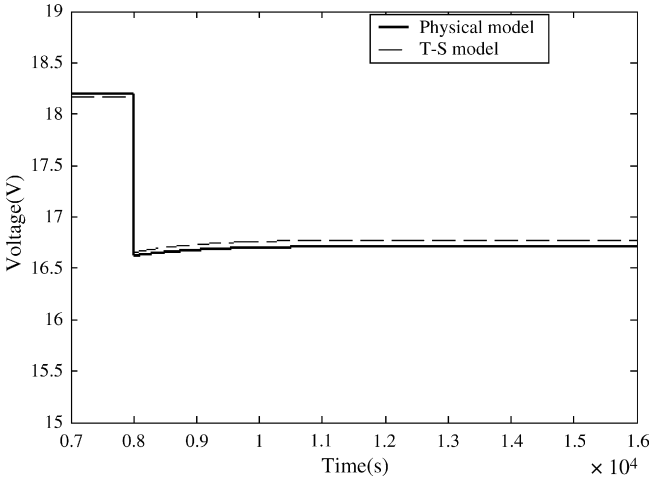

Fig. 5. Output voltage dynamic response.

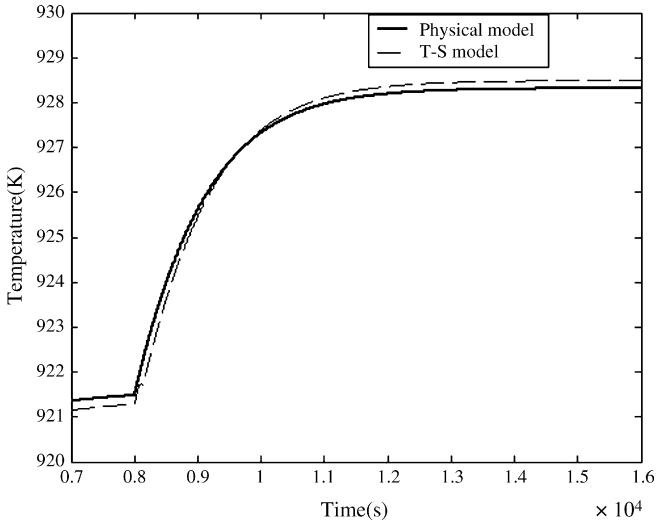

Fig. 6. Output temperature dynamic response.

5. Conclusions

To facilitate valid control strategy design and analysis of system stability, a fuzzy modeling study of MCFC is reported in this paper. An offline identification method for T–S fuzzy model is presented consists of the *if-part* identification and the *then-part* identification with a novel way to determine satisfactory number of fuzzy rules. It is shown that the identified MIMO T–S fuzzy model of MCFC is more attractive in that it avoids using complicated differential equations to describe the stack, and the inputs–outputs static and dynamic characteristics of a MCFC stack can be predicted. In the future, based on this T–S fuzzy model, some control scheme studies such as predictive control and robust control can be developed. In addition, an online identification algorithm for T–S fuzzy model with more variables can be considered.

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